1. As shown in the figure,
   The angle between \( \vec{A} \) and \( \vec{B} \) = \( 110^\circ - 20^\circ = 90^\circ \)
   \(|\vec{A}| = 3\) and \(|\vec{B}| = 4\)m
   Resultant \( \vec{R} = \sqrt{A^2 + B^2 + 2AB \cos \theta} = 5\) m
   Let \( \beta \) be the angle between \( \vec{R} \) and \( \vec{A} \)
   \[ \beta = \tan^{-1} \left( \frac{4 \sin 90^\circ}{3 + 4 \cos 90^\circ} \right) = \tan^{-1} \left( \frac{4}{3} \right) = 53^\circ \]
   \[ \therefore \text{Resultant vector makes angle } (53^\circ + 20^\circ) = 73^\circ \text{ with x-axis.} \]

2. Angle between \( \vec{A} \) and \( \vec{B} \) is \( \theta = 60^\circ - 30^\circ = 30^\circ \)
   \(|\vec{A}| \) and \(|\vec{B}| = 10\) unit
   \( R = \sqrt{10^2 + 10^2 + 2 \times 10 \times 10 \cos 30^\circ} = 19.3\) m
   \( \beta \) be the angle between \( \vec{R} \) and \( \vec{A} \)
   \[ \beta = \tan^{-1} \left( \frac{10 \sin 30^\circ}{10 + 10 \cos 30^\circ} \right) = \tan^{-1} \left( \frac{1}{2 + \sqrt{3}} \right) = \tan^{-1} (0.26795) = 15^\circ \]
   \[ \therefore \text{Resultant makes } 15^\circ + 30^\circ = 45^\circ \text{ angle with x-axis.} \]

3. \( x \) component of \( \vec{A} = 100 \cos 45^\circ = 100 / \sqrt{2} \) unit
   \( x \) component of \( \vec{B} = 100 \cos 135^\circ = 100 / \sqrt{2} \)
   \( x \) component of \( \vec{C} = 100 \cos 315^\circ = 100 / \sqrt{2} \)
   Resultant \( x \) component = \( 100 / \sqrt{2} - 100 / \sqrt{2} + 100 / \sqrt{2} = 100 / \sqrt{2} \)
   \( y \) component of \( \vec{A} = 100 \sin 45^\circ = 100 / \sqrt{2} \) unit
   \( y \) component of \( \vec{B} = 100 \sin 135^\circ = 100 / \sqrt{2} \)
   \( y \) component of \( \vec{C} = 100 \sin 315^\circ = -100 / \sqrt{2} \)
   Resultant \( y \) component = \( 100 / \sqrt{2} + 100 / \sqrt{2} - 100 / \sqrt{2} = 100 / \sqrt{2} \)
   Resultant = 100
   \[ \tan \alpha = \frac{y \text{ component}}{x \text{ component}} = 1 \]
   \[ \Rightarrow \alpha = \tan^{-1} (1) = 45^\circ \]
   The resultant is 100 unit at 45° with x-axis.

4. \( \vec{a} = 4\vec{i} + 3\vec{j} \), \( \vec{b} = 3\vec{i} + 4\vec{j} \)
   a) \[ |\vec{a}| = \sqrt{4^2 + 3^2} = 5 \]
   b) \[ |\vec{b}| = \sqrt{9 + 16} = 5 \]
   c) \[ |\vec{a} + \vec{b}| = |7\vec{i} + 7\vec{j}| = 7\sqrt{2} \]
   d) \[ \vec{a} - \vec{b} = (-3 + 4)\vec{i} + (-4 + 3)\vec{j} = \vec{i} - \vec{j} \]
   \[ |\vec{a} - \vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}. \]
5. x component of \( \overrightarrow{OA} = 2\cos30° = \sqrt{3} \)
\( x \) component of \( \overrightarrow{BC} = 1.5 \cos 120° = -0.75 \)
\( x \) component of \( \overrightarrow{DE} = 1 \cos 270° = 0 \)
\( y \) component of \( \overrightarrow{OA} = 2 \sin 30° = 1 \)
\( y \) component of \( \overrightarrow{BC} = 1.5 \sin 120° = 1.3 \)
\( y \) component of \( \overrightarrow{DE} = 1 \sin 270° = -1 \)
\( R_x = x \) component of resultant = \( \sqrt{3} - 0.75 + 0 = 0.98 \) m
\( R_y = \) resultant \( y \) component = \( 1 + 1.3 - 1 = 1.3 \) m
So, \( R = \) Resultant = 1.6 m
If it makes and angle \( \alpha \) with positive x-axis
\( \tan \alpha = y \) component \( x \) component
\( \Rightarrow \alpha = \tan^{-1} 1.32 \)

6. \( |\hat{a}| = 3 \) m \( |\hat{b}| = 4 \)
a) If \( R = 1 \) unit \( \Rightarrow \sqrt{3^2 + 4^2 + 2.3.4.\cos\theta} = 1 \)
\( \Rightarrow \theta = 180° \)
b) \( \sqrt{3^2 + 4^2 + 2.3.4.\cos\theta} = 5 \)
\( \Rightarrow \theta = 90° \)
c) \( \sqrt{3^2 + 4^2 + 2.3.4.\cos\theta} = 7 \)
\( \Rightarrow \theta = 0° \)
Angle between them is 0°.

7. \( \overrightarrow{AD} = 2\hat{i} + 0.5\hat{j} + 4\hat{k} = 6\hat{i} + 0.5\hat{j} \)
\( \overrightarrow{AD} = \sqrt{AE^2 + DE^2} = 6.02 \) KM

\( \tan \theta = DE / AE = 1/12 \)
\( \theta = \tan^{-1} (1/12) \)
The displacement of the car is 6.02 km along the distance \( \tan^{-1} (1/12) \) with positive x-axis.

8. In \( \triangle ABC \), \( \tan \theta = x/2 \) and in \( \triangle DCE \), \( \tan \theta = (2 - x)/4 \)
\( \Rightarrow 4 - 2x = 4x \)
\( \Rightarrow 6x = 4 \Rightarrow x = 2/3 \) ft

a) In \( \triangle ABC \), \( AC = \sqrt{AB^2 + BC^2} = \frac{2}{3}\sqrt{10} \) ft

b) In \( \triangle DCE \), \( DE = 1 - (2/3) = 4/3 \) ft
\( CD = 4 \) ft. So, \( CE = \sqrt{CD^2 + DE^2} = \frac{4}{3}\sqrt{10} \) ft

c) In \( \triangle AGE \), \( AE = \sqrt{AG^2 + GE^2} = 2\sqrt{2} \) ft.

9. Here the displacement vector \( \overrightarrow{r} = 7\hat{i} + 4\hat{j} + 3\hat{k} \)
a) magnitude of displacement = \( \sqrt{74} \) ft
b) the components of the displacement vector are 7 ft, 4 ft and 3 ft.
10. \( \vec{a} \) is a vector of magnitude 4.5 unit due north.
   
   a) \( 3|\vec{a}| = 3 \times 4.5 = 13.5 \)  
   
   \( 3 \vec{a} \) is along north having magnitude 13.5 units.
   
   b) \( -4|\vec{a}| = -4 \times 1.5 = -6 \) unit  
   
   \( -4 \vec{a} \) is a vector of magnitude 6 unit due south.
   
11. \( |\vec{a}| = 2 \text{ m} \), \( |\vec{b}| = 3 \) m  
   
   angle between them \( \theta = 60^\circ \)
   
   a) \( 3|\vec{a}||\vec{b}| = 3 \times 4.5 = 13.5 \) unit
   
   \( 3 \vec{a} \) is along north having magnitude 13.5 units.
   
   b) \( -4|\vec{a}||\vec{b}| = -4 \times 1.5 = -6 \) unit
   
   \( -4 \vec{a} \) is a vector of magnitude 6 unit due south.
   
12. We know that according to polygon law of vector addition, the resultant of these six vectors is zero.

   Here \( A = B = C = D = E = F \) (magnitude)

   So, \( R_x = A \cos \theta + A \cos \pi/3 + A \cos 2\pi/3 + A \cos 3\pi/3 + A \cos 4\pi/4 + A \cos 5\pi/5 = 0 \)

   [As resultant is zero. X component of resultant \( R_x = 0 \)]

   \( = \cos \theta + \cos \pi/3 + \cos 2\pi/3 + \cos 3\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0 \)

   Note: Similarly it can be proved that,

   \( \sin \theta + \sin \pi/3 + \sin 2\pi/3 + \sin 3\pi/3 + \sin 4\pi/3 + \sin 5\pi/3 = 0 \)

13. \( \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k} \); \( \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k} \)

   \( \vec{a} \cdot \vec{b} = ab \cos \theta = \cos^{-1} \frac{\vec{a} \cdot \vec{b}}{|ab|} \)

   \( \Rightarrow \cos^{-1} \frac{2 \times 3 + 3 \times 4 + 4 \times 5}{\sqrt{2^2 + 3^2 + 4^2 + 4^2 + 3^2}} = \cos^{-1} \left( \frac{38}{\sqrt{1450}} \right) \)

14. \( \vec{A} \cdot (\vec{A} \times \vec{B}) = 0 \) (claim)

   As, \( \vec{A} \times \vec{B} = AB \sin \theta \hat{n} \)

   \( AB \sin \theta \hat{n} \) is a vector which is perpendicular to the plane containing \( \vec{A} \) and \( \vec{B} \), this implies that it is also perpendicular to \( \vec{A} \). As dot product of two perpendicular vector is zero.

   Thus \( \vec{A} \cdot (\vec{A} \times \vec{B}) = 0 \).

15. \( \vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k} \), \( \vec{B} = 4\hat{i} + 3\hat{j} + 2\hat{k} \)

   \( \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 3 & 2 \end{vmatrix} = \hat{i}(6-12) - \hat{j}(8-12) + \hat{k}(6-12) = -6\hat{i} - 4\hat{j} - 6\hat{k} \).

16. Given that \( \vec{A} \), \( \vec{B} \) and \( \vec{C} \) are mutually perpendicular  

   \( \vec{A} \times \vec{B} \) is a vector which direction is perpendicular to the plane containing \( \vec{A} \) and \( \vec{B} \).

   Also \( \vec{C} \) is perpendicular to \( \vec{A} \) and \( \vec{B} \)

   . Angle between \( \vec{C} \) and \( \vec{A} \times \vec{B} \) is 0° or 180° (fig.1)

   So, \( \vec{C} \times (\vec{A} \times \vec{B}) = 0 \)

   The converse is not true.

   For example, if two of the vector are parallel, (fig.2), then also \( \vec{C} \times (\vec{A} \times \vec{B}) = 0 \)

   So, they need not be mutually perpendicular.
17. The particle moves on the straight line PP' at speed v.

From the figure,
\[ \overrightarrow{OP} \times \overrightarrow{v} = (\overrightarrow{OP})v \sin \theta \hat{n} = \overrightarrow{v}(\overrightarrow{OP}) \sin \theta \hat{n} = \overrightarrow{v}(\overrightarrow{OQ}) \hat{n} \]

It can be seen from the figure, \( \overrightarrow{OQ} = \overrightarrow{OP} \sin \theta = \overrightarrow{OP}' \sin \theta' \)

So, whatever may be the position of the particle, the magnitude and direction of \( \overrightarrow{OP} \times \overrightarrow{v} \) remain constant.

\[ \therefore \overrightarrow{OP} \times \overrightarrow{v} \text{ is independent of the position } P. \]

18. Given \( F = q \overrightarrow{E} + q(\overrightarrow{v} \times \overrightarrow{B}) = 0 \)

\[ \Rightarrow \overrightarrow{E} = - (\overrightarrow{v} \times \overrightarrow{B}) \]

So, the direction of \( \overrightarrow{v} \times \overrightarrow{B} \) should be opposite to the direction of \( \overrightarrow{E} \). Hence, \( \overrightarrow{v} \) should be in the positive yz-plane.

Again, \( E = v \overrightarrow{B} \sin \theta \Rightarrow v = \frac{E}{B \sin \theta} \)

For \( v \) to be minimum, \( \theta = 90^\circ \) and so \( v_{\min} = \frac{F}{B} \)

So, the particle must be projected at a minimum speed of \( \frac{E}{B} \) along +ve z-axis (\( \theta = 90^\circ \)) as shown in the figure, so that the force is zero.

19. For example, as shown in the figure,

\[ \overrightarrow{A} \perp \overrightarrow{B} \quad \overrightarrow{B} \text{ along west} \]
\[ \overrightarrow{B} \perp \overrightarrow{C} \quad \overrightarrow{A} \text{ along south} \]
\[ \overrightarrow{C} \text{ along north} \]

\[ \overrightarrow{A} \cdot \overrightarrow{B} = 0 \quad \therefore \overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{B} \cdot \overrightarrow{C} \]
\[ \overrightarrow{B} \cdot \overrightarrow{C} = 0 \quad \text{But } \overrightarrow{B} \neq \overrightarrow{C} \]

20. The graph \( y = 2x^2 \) should be drawn by the student on a graph paper for exact results.

To find slope at any point, draw a tangent at the point and extend the line to meet x-axis. Then find \( \tan \theta \) as shown in the figure.

It can be checked that,

\[ \text{Slope} = \tan \theta = \frac{dy}{dx} = \frac{d}{dx} (2x^2) = 4x \]

Where \( x \) = the x-coordinate of the point where the slope is to be measured.

21. \( y = \sin x \)

So, \( y + \Delta y = \sin (x + \Delta x) \)
\[ \Delta y = \sin (x + \Delta x) - \sin x \]
\[ = \left( \pi + \frac{\pi}{100} \right) - \sin \frac{\pi}{3} = 0.0157. \]

22. Given that, \( i = i_0 e^{-t/RC} \)

\[ \therefore \text{Rate of change of current} = \frac{di}{dt} = \frac{d}{dt} i_0 e^{-t/RC} = i_0 \frac{d}{dt} e^{-t/RC} = \frac{-i_0}{RC} e^{-t/RC} \]

When

a) \( t = 0, \frac{di}{dt} = \frac{-i}{RC} \)

b) when \( t = RC, \frac{di}{dt} = \frac{-i}{RC} \)

c) when \( t = 10RC, \frac{di}{dt} = \frac{-i_0}{RC e^{10}} \)
23. Equation \( i = i_0 e^{-t/RC} \)

\[
i_0 = 2A, \quad R = 6 \times 10^{-5} \Omega, \quad C = 0.0500 \times 10^{-6} F = 5 \times 10^{-7} F
\]

\( a \)
\[
i = 2 \times e^{\frac{(-0.3)}{6 \times 10^{-5} \times 5 \times 10^{-7}}} = 2 \times e^{(\frac{-0.3}{0.3})} = \frac{2}{e} \text{amp}.
\]

\( b \)
\[
\frac{di}{dt} = \frac{-i_0}{RC} e^{-t/RC} \quad \text{when } t = 0.3 \text{ sec} \Rightarrow \frac{di}{dt} = -\frac{2}{0.3} e^{-0.3/0.3} = \frac{-20}{3e} \text{Amp/sec}
\]

\( c \)
\[
\text{At } t = 0.31 \text{ sec, } i = 2e^{(-0.3/0.3)} = \frac{5.8}{3e} \text{ Amp}.
\]

24. \( y = 3x^2 + 6x + 7 \)

\( \therefore \text{Area bounded by the curve, x axis with coordinates with } x = 5 \text{ and } x = 10 \text{ is given by,} \)
\[
\text{Area} = \int_{5}^{10} (3x^2 + 6x + 7) \, dx = \left[ \frac{x^3}{3} + \frac{5x^2}{5} + 7x \right]_{5}^{10} = 1135 \text{ sq.units}.
\]

25. \[
\text{Area} = \int_{0}^{y} dy = \int_{0}^{\sin x} dx = -[\cos x]_{0}^{\pi} = 2
\]

26. The given function is \( y = e^{-x} \)

When \( x = 0, \ y = e^{0} = 1 \)

\( x \) increases, \( y \) value decreases and only at \( x = \infty, \ y = 0 \).

So, the required area can be found out by integrating the function from 0 to \( \infty \).

\[
\text{So, Area} = \int_{0}^{\infty} e^{-x} \, dx = -[e^{-x}]_{0}^{\infty} = 1.
\]

27. \( \rho = \frac{\text{mass}}{\text{length}} = a + bx \)

\( a \)
\( \text{S.I. unit of ‘a’ = kg/m and SI unit of ‘b’ = kg/m}^2 \text{ (from principle of homogeneity of dimensions)} \)

\( b \)
\( \text{Let us consider a small element of length ‘dx’ at a distance x from the origin as shown in the figure.} \)
\[\therefore \text{dm = mass of the element = } \rho \, dx = (a + bx) \, dx \]

\[
\text{So, mass of the rod} = m = \int_{0}^{L} (a + bx) \, dx = \left[ ax + \frac{bx^2}{2} \right]_{0}^{L} = aL + \frac{bL^2}{2}
\]

28. \[
\frac{dp}{dt} = (10 \text{ N}) + (2 \text{ N/S})t
\]

\( \text{momentum is zero at } t = 0 \)
\( \therefore \text{momentum at } t = 10 \text{ sec will be} \)
\[
\text{dp} = [(10 \text{ N}) + 2Ns] \, t dt
\]
\[
\int_{0}^{10} dp = \int_{0}^{10} 10 dt + \int_{0}^{10} (2t dt) = 10t_{0}^{10} + 2 \frac{t^2}{2} = 200 \text{ kg m/s}.
\]
29. The change in a function of $y$ and the independent variable $x$ are related as $\frac{dy}{dx} = x^2$.

$\Rightarrow dy = x^2 \, dx$

Taking integration of both sides,

$$\int dy = \int x^2 \, dx \Rightarrow y = \frac{x^3}{3} + c$$

$\therefore y$ as a function of $x$ is represented by $y = \frac{x^3}{3} + c$.

30. The number significant digits

a) 1001 No. of significant digits = 4
b) 100.1 No. of significant digits = 4
c) 100.10 No. of significant digits = 5
d) 0.001001 No. of significant digits = 4

31. The metre scale is graduated at every millimeter.

$1 \, m = 100 \, mm$

The minimum no. of significant digit may be 1 (e.g. for measurements like 5 mm, 7 mm etc) and the maximum no. of significant digits may be 4 (e.g. 1000 mm).

So, the no. of significant digits may be 1, 2, 3 or 4.

32. a) In the value 3472, after the digit 4, 7 is present. Its value is greater than 5.

$\therefore$ the next two digits are neglected and the value of 4 is increased by 1.

$\therefore$ value becomes 3500

b) value = 84

c) 2.6

d) value is 28.

33. Given that, for the cylinder

Length $l = l = 4.54 \, cm$, radius $r = 1.75 \, cm$

Volume $V = \pi r^2 l = \pi \times (4.54) \times (1.75)^2$

Since, the minimum no. of significant digits on a particular term is 3, the result should have 3 significant digits and others rounded off.

So, volume $V = \pi r^2 l = (3.14) \times (1.75) \times (1.75) \times (4.54) = 43.6577 \, cm^3$

Since, it is to be rounded off to 3 significant digits, $V = 43.7 \, cm^3$.

34. We know that,

Average thickness $\frac{2.17 + 2.17 + 2.18}{3} = 2.1733 \, mm$

Rounding off to 3 significant digits, average thickness $= 2.17 \, mm$.

35. As shown in the figure,

Actual effective length $= (90.0 + 2.13) \, cm$

But, in the measurement 90.0 cm, the no. of significant digits is only 2.

So, the addition must be done by considering only 2 significant digits of each measurement.

So, effective length $= 90.0 + 2.1 = 92.1 \, cm$.

* * * *